

Unit 9  
Day 4  
Variation

### Direct Variation:

If  $y$  varies directly as the  $n$ th power of  $x$  **or** if  $y$  is directly proportional to the  $n$ th power of  $x$ , then  $y=kx$ . Where  $k$  is the constant of variation (proportionality).

Ex1: Write the equation  
r is directly proportional to a with a constant of  
variation of 16.

$$r = 16a$$

Ex2: Write the equation  
a varies directly with b with a constant of variation of  $\frac{3}{5}$ .

$$k = \frac{3}{5}$$

$$a = \frac{3}{5}b$$

### Inverse Variation:

If  $y$  varies inversely as the  $n$ th power of  $x$ , **or** If  $y$  is inversely proportional to the  $n$ th power of  $x$ , then  $y = \frac{k}{x^n}$ .

Ex1: Write the equation

If b varies inversely as r with a constant variation of 2.6.

$$b = \frac{2.6}{r}$$

Ex2: Write the equation:  $y$  varies inversely as  $x^3$ .

$$y = \frac{k}{x^3}$$

### Joint Variation:

If  $y$  varies jointly with the  $n$ th power of  $x$  and  $z$ , then  
 $y = kx^nz$ .



Ex1: Write the equation  
w varies jointly with c and  $f^2$  with a constant of  
proportionality  $\pi$ .

of

$$W = c f^2 \pi$$

$$\underline{W = \pi c f^2}$$

Ex2: Write the equation

D varies jointly with r and t with a constant of variation of 1.

$$D = rt$$

Ex3: Write the equation  
r is proportional to  $x^2$  and inversely proportional to y.

$$r = \frac{kx^2}{y}$$

Write each formula as an English phrase.

Ex1:

$C=2\pi r$ , where  $c$  is the circumference of a circle of radius  $r$ .

The circ. varies directly w/  
 $r$  w/ a COV of  $2\pi$

Write each formula as an English phrase.

Ex2:

$r = \frac{d}{t}$ , where  $r$  is the average speed when traveling  $d$  miles in  $t$  hours.

rate( $r$ ) varies directly with  
distance ( $d$ ) and indirectly  
with time ( $t$ )

Write each formula as an English phrase.

Ex:3

$s=kx^3$ , where  $s$  is the strength of a muscle that has length  $x$ .

Strength ( $s$ )  
~~is~~ is directly prop. to  
the ~~cube~~ of the length of a muscle ( $x$ )  
~~with~~ with a constant of  
variation of  $k$ .

Solve each variation problem:

Ex1:

If  $m$  varies directly as  $x$  and  $y$ , and  $m = 10$  when  $x = 4$  and  $y = 7$ , find  $m$  when  $x = 11$  and  $y = 8$

$$m = kxy$$
$$10 = k(4)(7)$$

$$10 = 28k$$

$$k = \frac{10}{28} = \frac{5}{14}$$

$$m = \frac{5}{14}xy$$

$$m = \frac{5}{14}(11)(8)$$

$$m = \frac{220}{7}$$

Ex2:

What happens to  $y$  if  $y$  varies inversely as  $x$ , and  $x$  is doubled?

$$y = \frac{k}{x}$$

$$y = \frac{k}{2x} \quad \begin{array}{l} \text{double} \\ x \end{array}$$

$y$  is cut in half.



Ex3:

What happens to  $A$  if  $A$  varies jointly with  $b$  and  $h$  if  $b$  is quartered and  $h$  is doubled?

$$A = kbh$$

$$A = k\left(\frac{b}{4}\right)(2h)$$

$$A = \frac{k.bh}{2} \quad A \text{ is halved.}$$

Lets apply what we have learned.

The pressure of a compressed gas is inversely proportional to its volume according to Boyle's Law. A pressure of 40 pounds per square inch is created by 600 cubic inches of a certain gas. Find the pressure when the gas is compressed to 200 cubic inches.

$$\begin{aligned} \text{let } p &= \text{pressure (p/in}^2\text{)} \\ V &= \text{volume (in}^3\text{)} \\ p &= \frac{k}{V} & p &= \frac{24,000}{V} \\ 40 &= \frac{k}{600} & p &= \frac{24000}{200} \\ k &= 24,000 & p &= 120 \text{ lbs/in}^2 \end{aligned}$$

Hooke's law for an elastic spring states that the distance a spring stretches varies directly as the force applied. A force of 30 pounds stretches a certain spring 5 inches. Find how far the spring is stretched by a 50 pound weight.

$$\begin{aligned} \text{let distance} &= d(\text{in}) \\ \text{force} &= f \text{ (lbs)} \end{aligned}$$

$$d = k f$$

$$5 = 30 k$$

$$k = \frac{1}{6}$$

$$d = \frac{25}{3} = 8\frac{1}{3} \text{ in}$$

$$d = \frac{1}{6} f$$

$$d = \frac{1}{6}(50)$$

HW pg 334-336 2-6, 12-40 even 43,44 all